

The role of gas dynamical friction in the evolution of embedded stellar clusters

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ABSTRACT

Two puzzles associated with open clusters have attracted a lot of attention – their formation, with densities and velocity dispersions that are not too different from those of the star forming regions in the Galaxy, given that the observed Star Formation Efficiencies (SFE) are low and, the mass segregation observed / inferred in some of them, at ages significantly less than the dynamical relaxation times in them. Gas dynamical friction has been considered before as a mechanism for contracting embedded stellar clusters, by dissipating their energy. This would locally raise the SFE which might then allow bound clusters to form. Noticing that dynamical friction is inherently capable of producing mass segregation, since here, the dissipation rate is proportional to the mass of the body experiencing the force, we explore further, some of the details and implications of such a scenario, vis-a-vis observations. Making analytical approximations, we obtain a boundary value for the density of a star forming clump of given mass, such that, stellar clusters born in clumps which have densities higher than this, could emerge bound after gas loss. For a clump of given mass and density, we find a critical mass such that, sub-condensations with larger masses than this could suffer significant segregation within the clump.

Key words. open clusters and associations: general - galaxies: star clusters: general - stars: kinematics and dynamics

1. Introduction

Stellar clusters can be viewed out to far distances and hence are important probes for our study of the universe. Most stars seem to form in clusters, and hence, understanding clusters becomes an essential requirement for understanding the formation and evolution of galaxies. Two puzzles associated with open clusters that have attracted a lot of attention are – their formation, with densities and velocity dispersions that are not too different from those of the star forming regions in the Galaxy, given that the observed Star Formation Efficiencies (SFE) are low (Tutukov 1978; Hills 1980; Mathieu 1983; Elmegreen 1983; Lada, Margulis & Dearborn 1984; Adams 2000; Geyer & Burkert 2001; Baumgardt & Kroupa 2007; Goodwin 2009) and, the mass segregation observed / inferred in some of them (de Grijs et al. 2002; Sirianni et al. 2002; Gouliermis et al. 2004; Chen, de Grijs & Zhao 2007; Converse & Stahler 2010; Hasan & Hasan 2011), at ages significantly less than their dynamical relaxation times (Bonnell & Davies 1998; McMillan, Vesperini & Portegies Zwart 2007; Allison et al. 2009, 2010; Maschberger & Clarke 2011; Olczak, Spurzem & Henning 2011). In this paper, by making analytical approximations, we explore the efficacy of gas dynamical friction, operating during the embedded phase of stellar clusters, in solving these two puzzles.

Star clusters form, hidden from sight, embedded in the dense, dark and cold cores of giant molecular clouds (Lada & Lada 2003). A star forming gas cloud converts less than ten percent of its mass into stars and then, in less than ten million years (Palla & Stahler 2000) loses the gas which had been gravitationally binding the stellar cluster. A cluster of stars will be bound if the total energy of the stars is negative (Chandrasekhar & Elbert 1972). For mass loss that is instantaneous, analytical approximations (Hills 1980) show that the Star Formation Efficiency (SFE), i.e. the fraction of the total mass of the gas cloud that is converted into stars, should be greater than 0.5, if a bound cluster is to emerge after gas loss. On the other hand, if the mass loss is gradual, bound clusters similar to those observed could emerge for SFE's greater than ~ 0.3 (Mathieu 1983; Elmegreen 1983; Lada, Margulis & Dearborn 1984). Note however that this value is still much higher than the observed values for the global SFE. The SFE in dense cores which are forming clusters seem to be $\sim 0.2 - 0.3$, while the observed values for the SFE in star forming clouds are an order of magnitude smaller (Lada & Lada 2003).

Observations show also that, embedded clusters can start showing mass segregation in less than a few million years (Schmeja, Kumar & Ferreira 2008) and that the Orion Nebula Cluster is not old enough for the observed segregation of its massive stars to be due to dynamical relaxation by two body encounters alone (Bonnell & Davies 1998). Simulations

indicate that the Pleiades cluster started out with significant mass segregation in the embedded phase itself, with high mass stars preferentially concentrated towards the center (Converse & Stahler 2010). The dynamical relaxation time in Pleiades is twice the present age of the cluster. Also, at the same time, some clusters like Taurus and Trumpler 16 do not show mass segregation (Parker et al 2011; Wolk et al. 2011; Hasan & Hasan 2011).

Numerical investigations of the dependence of the survivability of a cluster on the Star Formation Efficiency has shown that, a significant fraction of the stars can remain bound, after rapid gas loss, for an SFE $\gtrsim 0.3$ and, for slow gas removal, up to 50% of the cluster can remain bound for SFE values down to 0.15 – 0.2 (Geyer & Burkert 2001; Baumgardt & Kroupa 2007; Goodwin 2009). However, if most open clusters are remnants of embedded clusters that have – consequent to gas removal – lost one half or more of their natal members (Bastian & Goodwin 2006; Goodwin & Bastian 2006), the similarity in the mass functions of embedded and open clusters and, the similarity in the IMF's of embedded and open clusters become difficult to understand (Lada & Lada 2003). In this context, scenarios in which bound clusters can form, without losing too much of their natal stellar content and, with (possibly) mass segregation become interesting. Gas dynamical friction due to an embedding gaseous medium has been invoked, in understanding many phenomena; at various scales – from that of planetary systems to that of clusters of galaxies (Kim & Kim 2009). In the case of stars, while considering mechanisms that change the momenta of stars in stellar systems, Chumak & Chumak (1976) conclude that, dynamical friction in a dust cloud would be the dominant momentum changing mechanism for a star, when there are no massive scattering centres and the speed of the star does not exceed a limit. Making use of the Chandrasekhar dynamical friction formula (Chandrasekhar 1943,a), Saiyadpour, Deiss & Kegel (1997) had explored gas dynamical friction as a means to produce contraction of embedded clusters, thus raising the SFE locally, which could then lead to the formation of bound clusters. They had suggested also that, the same might be responsible for mass segregation (Deiss et al 1998). Gorti & Bhatt (1995) had explored the effect of gas dynamical friction on pre-stellar clumps by numerical simulations and also, it has been noticed that gas dynamical friction can reduce the time scale for dynamical mass segregation (Er et al. 2009). However, no general analysis of such a scenario, or its details and implications vis-a-vis observations, has been made. Here, using analytical approximations, we explore a scenario in which gas dynamical friction could be playing a significant role, in the early evolution of embedded stellar clusters, as the chief mechanism which can cause them to contract and form more strongly bound configurations and, also cause mass segregation in them. We check the plausibility of the model by examining its consistency with observations.

In section 2, using simple virial considerations, we check that, a scenario in which embedded clusters undergo contraction, meets the dynamical constraints set by observations. In section 3 we consider dynamical friction in star forming gas clouds. By requiring that the gas dynamical friction time scale within an embedded cluster be less than the time that is available before gas is expelled from the cluster, we obtain a boundary value for the density of a star forming clump of given mass, such that, stellar clusters born in clumps which have densities higher than this, could emerge bound after gas loss, and for a clump of given mass and density, we find a critical mass such that, subcondensations with larger masses than this could suffer significant segregation within the clump. In section 4 we compare our results with observations. In section 5 we discuss some of the possible implications of our scenario and, their accord with observations. In section 6 we give a brief summary of our results.

2. The SFE as a constraint on cluster formation scenarios -analytical approximations

We now proceed as follows, for checking, whether the scenario we propose, wherein a nascent cluster contracts within the parent gas cloud, meets the constraints set by observations. For a gas cloud that converts a fraction ϵ of its mass into stars and then loses the remaining gas, Hills (1980) had, by simple virial considerations, obtained expressions for the ratios of, the radius (r_0), velocity dispersion (σ_0) and density (ρ_0) of the gas cloud, with the radius (r_f), velocity dispersion (σ_f) and density (ρ_f) of the stellar cluster, assumed virialized after gas loss (see also (Mathieu 1983)), as $\sigma_f/\sigma_0 = \sqrt{\epsilon/\sqrt{r_f/r_0}}$ and $\rho_f/\rho_0 = \epsilon/(r_f/r_0)^3$, where r_f/r_0 equals $\epsilon/(2\epsilon - 1)$ for instantaneous gas loss and equals $1/\epsilon$ for adiabatic gas loss. Making a similar analysis for an embedded stellar cluster, with $\epsilon < 0.5$, that contracts to a radius $r' = pr_0$, where p the contraction factor is < 1 we get, $r_f/r_0 = (p/(1 - ((1 - \epsilon)/\epsilon)p^3))$ for instantaneous gas loss and $r_f/r_0 = p(1 + ((1 - \epsilon)/\epsilon)p^3)$, if the gas is lost adiabatically.

For the first two cases, we see from the corresponding expressions for r_f/r_0 that, in the case of instantaneous mass loss, bound clusters may form only for SFE's > 0.5 and in the case of slow mass loss, bound clusters may form, technically, for all values of $\epsilon > 0$. In the case where the embedded cluster contracts to within a radius $r' = pr_0$, we see that, even if gas loss is instantaneous, bound clusters may form, for all values of $\epsilon < 0.5$ also, if $p < ((\epsilon/(1 - \epsilon))^{1/3})$. For $p \leq ((\epsilon/(1 - \epsilon))^{1/3})$, the contracted cluster will have an *apparent* SFE ≥ 0.5 (Saiyadpour, Deiss & Kegel 1997).

In a real situation, a cluster will be situated in a galaxy, and hence, will be able to retain only the stars that are within the tidal radius. Our analysis, thus may be applied to those clusters, that do not fill their tidal radii and also, are young enough for their dynamics to have not been affected by either internal processes, like mass loss from the stars or external processes like interactions with molecular clouds. The typical value for the density of a star forming region is $\sim 10^4 \text{ cm}^{-3}$ (Elmegreen 1983; Bergin & Tafalla 2007). And, taking for typical clusters, a density enhancement of 50 with respect to the density of field stars – which is of the order of $4 \times 10^{-24} \text{ g cm}^{-3}$ (Cox 1999) – we get a typical value for $\rho_f/\rho_0 \sim 0.005$. The typical value for the observed velocity dispersion for clusters is $\sim 0.5 \text{ km s}^{-1}$ and for molecular

clouds it is of the order of a few kilometer per second, giving a typical value for $\sigma_f/\sigma_0 \sim 1/4 - 1/6$.

We now estimate a typical value for the contraction factor p , by obtaining an estimate for r_f/r_0 using observations as, $r_f/r_0 = 1.7(\epsilon/0.025)^{1/3}((\rho_f/\rho_0)/0.005)^{-1/3}$ and then, solving for p using the relation $r_f/r_0 = (p/(1 - ((1 - \epsilon)/\epsilon)p^3))$ (in the case of instantaneous gas loss) or, $r_f/r_0 = p(1 + ((1 - \epsilon)/\epsilon)p^3)$ (in the case of adiabatic gas loss). The estimate for p may be used to calculate $\epsilon_a = 1/(1 + ((1 - \epsilon)/\epsilon)p^3)$, the *apparent* SFE of the cluster, when it has contracted to such an extent that, if it loses the gas now, it can remain bound, with a radius r_f (after virialization), such that $r_f/r_0 = 1.7$. For $r_f/r_0 = 1.7$ and $\epsilon = 0.025$, we get the required values for the contraction factor p as, $p = 0.278$ in the case of fast gas loss and $p = 0.426$ in the case of slow gas loss. These values for p , correspond respectively to *apparent* SFE's ~ 0.544 and 0.25 for the embedded stellar cluster, in its most contracted phase, just prior to losing the gas.

In our scenario, we would expect at least some embedded stellar clusters, to show such high (*apparent*) SFE's, when near to the end of their embedded phase. Stellar densities that imply SFE's upto 0.47 have been reported, in the dense embedded clusters observed in the ρ Ophiucus, IC348 and Orion-Trapezium star forming regions (Wilkings & Lada (1983) and references therein). We interpret these, as *apparent* SFE's, arising from the contraction of the particular nascent cluster. Thus these high SFE's that have been observed, lend support to our scenario.

As ϵ is varied from 0.008 to 0.08 the estimates for the appropriate *apparent* SFE's, as given by our model, go from 0.54 to 0.576 for fast gas loss and, from ~ 0.2 to 0.3 for slow gas loss. Assuming that r_f/r_0 has a larger value, will make the *apparent* SFE values that correspond to it smaller, and vice versa. As an aside we notice that, the value for the ratio σ_f/σ_0 , calculated from the observed value for the ratio of the densities as, $\sigma_f/\sigma_0 = 1/8(\epsilon/0.025)^{1/3}((\rho_f/\rho_0)/0.005)^{1/6}$ is consistent with its value determined independently from direct observations – a reflection of the fact that molecular clouds, as well as bound clusters, are both roughly in virial equilibrium, a condition which was assumed in deriving these relations.

Thus, we see that, our results linking r_f/r_0 , σ_f/σ_0 and ρ_f/ρ_0 , obtained under the assumption that the stellar cluster contracts within the parent gas cloud, are broadly consistent with observations. This conclusion of ours, is not very sensitive to the precise value of ϵ used in the calculations, as long as it is low. With the above conclusion as our motivation, we now explore, by making analytical approximations, the efficacy of gas dynamical friction, in bringing about such a contraction of embedded stellar clusters.

3. The gas dynamical friction time-scale

Molecular clouds which are observed to be the sites of star and cluster formation, have a hierarchical structure, of dense cores within clumps, the clumps themselves being within clouds – the density getting lower as the scale becomes larger, from cores to clumps to clouds (see for example (Bergin & Tafalla 2007)). For a clump of mass m , and velocity \mathbf{v} , moving in a medium with density ρ_{gas} , the gas dynamical friction time scale t_{gdf} is $\sim (E/(dE/dt))$, where dE/dt is the rate of energy dissipation due to gas dynamical friction and E is the kinetic energy of the clump.

The retarding force, when a massive perturber that is interacting gravitationally moves through a fluid, may be determined by considering the interaction of the body with its own gravitationally induced over-density wake. For density wakes with small amplitudes, Ostriker (1999) using time-dependent linear perturbation theory, determined the drag force on a point mass perturber, moving in a straight line, through a uniform, infinite, gaseous medium. For a mass m , moving with a speed v , in a medium with density ρ_{gas} and sound speed c_s , she obtained an expression for the force as

$$\mathbf{F}_{Lin} = -\frac{4\pi\rho_{gas}(Gm)^2}{v^3}\mathbf{v}f(M) \quad (1)$$

where $f(M) = 1/2\ln((1+M)/(1-M)) - M$ for $M < 1$ and $= 1/2\ln(1 - (1/M^2)) + \ln(vt/r_{min})$ for $M > 1$. Here $M = v/c_s$ is the Mach number, t is the time for which the perturber has been moving in the medium, and r_{min} is a minimum radius introduced to avoid singularity in the force evaluation. Later workers have relaxed the constraint on the perturber being point-like, to a perturber which has no boundary but is merging into the surrounding medium; that of a linear trajectory to considering circular orbits; and that of a uniform medium to one that is radially stratified (Kim & Kim 2009 and references therein). These authors found that equation (1) is generally applicable with appropriate changes to the factor (vt/r_{min}) .

Thus we get

$$\frac{dE}{dt} = k\mathbf{F}_{Lin} \cdot \mathbf{v} = k\frac{4\pi\rho_{gas}(Gm)^2}{v}f(M), \quad (2)$$

where k is a numerical factor that accounts for departures from the linear theory. For a clump, moving within a cloud of total mass M_c and radius R , with a speed v , parametrized by the virial speed for the cloud as $v = \beta v_{virial}$, putting $\rho_{gas} = \epsilon_{gas}\rho$ where, the gas fraction $\epsilon_{gas} = M_{gas}/M_c$, M_{gas} being the mass in the interclump medium and ρ is

$M_c/(4\pi/3)R^3$, we get

$$t_{gdf} = \frac{\beta^3 v_{virial}^3 R^3}{6kG^2 m \epsilon_{gas} M_c f(M)}. \quad (3)$$

Real cores, clumps and clouds are not spherically symmetric, have density gradients, and also substructure at a variety of scales. However, potentials are rounder and smoother than the underlying density distribution (Baumgardt & Kroupa 2007), and relaxing the assumption of sphericity, or homogeneity, introduces only changes by factors of order unity in the various terms of the virial equation (Chandrasekhar & Elbert 1972; Som Sunder & Kochhar 1985; Verschueren 1990). Hence, for the term v_{virial} , we use the expression for the virial speed for a smooth homogeneous sphere of mass M_c and radius R viz. $\sqrt{(3/5)(GM_c/R)}$, and write

$$t_{gdf} = \frac{1}{20} \sqrt{\frac{9}{5\pi}} \frac{\beta^3 (M_c/m)}{k \epsilon_{gas} f(M)} t_{cross} = 0.038 \frac{\beta^3 (M_c/m)}{k \epsilon_{gas} f(M)} t_{cross}, \quad (4)$$

where $t_{cross} = (G\rho)^{-1/2}$ is the crossing time in the cloud.

We rewrite the expression for the gas dynamical friction time-scale (Eqn.4) as

$$t_{gdf} = \tau \mu t_{cross} \quad (5)$$

where

$$\tau = \frac{0.038 \beta^3}{k \epsilon_{gas} f(M)} \quad (6)$$

and the mass ratio factor μ , is given by

$$\mu = \frac{M_c}{m}. \quad (7)$$

3.1. Production of bound clusters and the occurrence of mass segregation

The chief effect of gas dynamical friction is to decelerate objects to sonic speeds. In molecular clouds, turbulence has been observed to be dominated by large modes, whereas sound speeds are rather low and subvirial (see for example Kirk, Johnstone & Tafalla (2007); Kirk et al (2010)). We may expect that a cluster will be bound after gas loss, if typical stars – i.e. those around the peak of the IMF, which carry most of the mass in a cluster – can lose energy in a time shorter than the gas expulsion time-scale t_E (Gieles 2010). In our case this would ensure that all higher mass objects would also be bound.

Thus, in our scenario, with $t_{cross} \sim (G\rho)^{-1/2}$, we get the condition for the formation of a bound cluster as

$$15\tau\mu(\rho_{M_\odot pc^{-3}})^{-1/2} < t_{E(Myr)}. \quad (8)$$

This yields a critical density for the parent gas cloud, such that the cluster emerges bound after gas loss, as

$$\rho > \rho_{critical} = 225(\tau\mu/t_{E(Myr)})^2 M_\odot pc^{-3}. \quad (9)$$

For given ρ , τ and M_c , we may rewrite the above inequality to give a critical mass $m_{critical}$ such that, all objects with

$$m > m_{critical} = 15(\tau M_c/t_{E(Myr)})(\rho_{M_\odot pc^{-3}})^{-1/2} \quad (10)$$

would be significantly slowed down by gas dynamical friction.

3.1.1. Estimates for the various factors

For a pre-cluster, we take t_E , the length of time for the pre-stellar embedded phase (Leisawitz, Bash & Thaddeus 1989), when gas dynamical friction would be operative (Sanchez-Salcedo & Brandenburg 2001), as 5 Myr. This is less than a quarter of the pre-main sequence contraction time for a one solar mass star (Charbonnel et al. 1999). Stellar winds, which can push away the ambient gas, thus reducing gas dynamical friction, are initiated when the stars reach the main sequence. Molecular outflows which have the same effect, have dynamical time scales that are much smaller, of the order of 0.1 Myr only (Frank 1999).

We may expect a bound cluster to form if cores with masses down to the order of $1 M_\odot$, which can produce stars with masses up to the peak of the IMF (Larsen 1982; Weidner & Kroupa 2006; Weidner, Kroupa & Bonnel 2010), which is $\approx 1/3 M_\odot$ (Chabrier 2003), have $t_{gdf} < 5$ Myr. This is because they will now have ample time to get decelerated to sub-virial speeds by gas dynamical friction, even as they migrate to the minimum of the gravitational potential.

We now make an estimate for τ , for star forming cores in clumps, and for clumps in molecular clouds. We first consider $f(M)$. The observed density and velocity structures in molecular clouds are consistent with supersonic turbulence driven at the scale of the cloud itself (Ossenkopf & McLow 2002). In determining the Mach number M for a perturber moving through a turbulent medium, the speed of small scale turbulence may be used instead of the sound speed, as the speed with which pressure perturbations are transmitted through the medium (Saizyadpour, Deiss & Kegel 1997). From Ostriker (1999) we see that, from $\sim M^3/3$ for $M \ll 1$, $f(M)$ rises sharply to $\sim \ln(vt/r_{min}) - 2$ across $M = 1$ and tends to $\ln(vt/r_{min})$ for $M \gg 1$. We see that the time scale will be shorter for motion that is supersonic, since $f(M)$ is > 1 only for supersonic motion (Ostriker 1999). It will be shorter for objects that have comparatively smaller speeds, since a smaller value for β , implies, both a smaller amount of energy that need be dissipated, and a smaller separation between the perturber and its wake. It depends on the density of the parent cloud via the crossing time, and is shorter for denser clouds. It decreases as the gas fraction in the cloud increases. Also, it depends directly on the mass of the parent cloud – the amount of energy that has to be dissipated will be proportional to M_c , since the virial speed is proportional to $\sqrt{M_c}$ – and inversely on the mass of the perturber, which decides the dissipation rate. It depends inversely on $f(M)$, which is related to the speed with which pressure forces can redistribute the density enhancement in the wake.

We consider now a distribution of speeds for the particles. Those objects which have supersonic speeds with respect to the large scale streaming motion of the gas surrounding it, can be significantly slowed down, by gas dynamical friction. For subsonic objects, the gas dynamical friction force is depressed due to the fact that pressure forces are able to restore the density distribution about the perturber, sooner (Ostriker 1999). We see that the chief effect of gas dynamical friction would be to slow supersonic objects down to sonic speeds. Super-virial objects are likely to escape from the system. So, from here onwards, we put $\beta = 1$ in the expression for t_{gdf} .

We now consider k , the correction factor for non-linearity. Kim & Kim (2009) investigating the gas dynamical friction force in the non-linear regime, by high resolution hydrodynamic simulations, had measured the strength of the induced gravitational perturbations, by a dimensionless parameter $A = (r_B/r_s) = (Gm/c_s^2 r_s)$ (r_B is the Bondi radius). A , which corresponds roughly to the ratio of the perturbed density at a distance r_s from the perturber, to the background density, is $\gg 1$ in the non-linear regime. Following them, in our case, we determine the correction necessitated by non-linearity, as follows. For a typical protostellar object of mass $1M_\odot$, taking $r_s \sim 100\text{AU}$ – the radius of a protostellar nebula, and with $c_s \sim 1\text{kms}^{-1}$, which is the order of the speed of sound/ random motions in molecular clouds, we get

$$A = 900(\text{kms}^{-1}/c_s)^2 (m/M_\odot)(\text{AU}/r_s) \quad (11)$$

to be ~ 9 . Kim & Kim (2009) obtain k in terms of a non-linearity parameter $\eta = (A/(M^2 - 1))$ as, $k = (F/F_{Lin}) = (\eta/2)^{-0.45}$ where F is the gas dynamical friction force in the non-linear regime. We convert the limiting values they obtain for η , into limits on the Mach number M for the protostellar object in a star forming gas cloud, as follows. For $M < 1$ and for $M > 3.7$ (i.e. $\eta < 0.7$) $(F/F_{Lin}) \sim 1$ and for $3.7 > M > 2.35$ (i.e. $0.7 < \eta < 2$), $(F/F_{Lin}) \sim 1.6 - 1.0$. For $2.35 > M > 1.04$ (i.e. $2 < \eta < 100$), $(F/F_{Lin}) \sim 1 - 1/5$. We see that for cores which are supersonic, but not greatly so, non-linearity introduces a reduction. For this range of speeds, we get a typical value for this reduction factor by evaluating it at $M = 2$ as $k = (\eta/2)^{-0.45} \sim 0.83$. For clumps, which are not compact like cores – with typical masses $\sim 100M_\odot$ and sizes \sim few light-year – we are in the linear regime, and $k = 1$. Thus we have $k \sim 0.83$ and $\epsilon_{gas} \sim 0.9$ for cores in clumps and, $k = 1$ and $\epsilon_{gas} \sim 0.75$ for clumps embedded in molecular clouds. Here the gas fraction within clouds has been taken as the complement of the mass fraction for clumps in clouds (Lada & Lada 2003; Kauffmann et al 2009).

For a perturber, which is moving in a turbulent medium, the wake would not be fully developed. Taking $\ln(vt/r_{min}) \sim 3$, i.e. for a size of the wake ~ 20 times the size of the perturber, $f(M) \sim 1$, and we find that τ is ~ 0.05 . This may be compared with the value $1/4\pi \approx 0.08$ obtained by Ostriker (1999), as the value for the coefficient relating the gas dynamical friction time scale to, the mass ratio and the typical time scale in the system, while considering the decay of the near-circular orbit of a massive perturber, moving in a massive, spherical cloud with a singular isothermal sphere profile, which result, was itself found consistent with the results obtained by Sanchez-Salcedo & Brandenburg (2001), in three dimensional simulations, of a gravitational perturber moving in a gaseous sphere.

4. Comparison with observations

The formula we have obtained above, for the critical density $\rho_{critical}$, such that, if the cloud gas density ρ is $> \rho_{critical}$, gas dynamical friction would be significant, for clumps of mass m , moving in a cloud of mass $M_c = m/\mu$, may be used to investigate the degree to which gas dynamical friction is significant for cores in clumps and for the clumps themselves, embedded in molecular clouds.

For a typical clump of mass $500M_\odot$ – which, given a SFE ~ 0.1 (Lada & Lada 2003), could be the progenitor of a typical cluster of mass $50M_\odot$ – we get a critical density $\sim 1.2 \times 10^5 \text{cm}^{-3}$. This is an order of magnitude higher than the density of typical clumps (Bergin & Tafalla 2007), which have densities $\sim 10^3 - 10^4 \text{cm}^{-3}$. Also we get the critical mass in typical clouds which have masses $10^3 - 10^4 M_\odot$ and densities $50 - 500 \text{cm}^{-3}$ as, $30 < m_{cr} < 10^3 M_\odot$, the lower value being

for less massive clouds with a higher density and vice versa. Cores which have $m > m_{cr}$, may lose significant amounts of energy, and could congregate (see also Kirk et al (2010)) at the potential minima, raising the apparent SFE there. The compact cluster that forms, can then emerge bound after gas expulsion, if the local SFE, gets raised sufficiently (see discussion before). Thus clusters which were born in sufficiently dense clumps, could emerge bound, and also show mass segregation (since protostars having higher masses and hence, comparatively lower values for μ , would be more strongly affected by gas dynamical friction), while those born in clumps with comparatively lower density, would form associations. For a clump of mass $10^3 M_\odot$, $\rho_{critical}$ is $\sim 5 \times 10^5 \text{ cm}^{-3}$; i.e. an order of magnitude larger, than densities in the upper range of values for typical clumps.

As a particular example, we now compare the predictions of our theory, with observations of the nearest and well studied Orion Nebula Cluster, which is young enough for the mass segregation observed in it to have been very fast (Reggiani et al 2011). It is observed that only stars with masses $\gtrsim 5 M_\odot$ are segregated in the ONC (Hillenbrand & Hartmann 1998). Huff & Stahler (2006) estimate the mass of the parent cloud of the ONC as $6700 M_\odot$, within a radius 2.5 pc, and we get the critical mass (Eqn 9), for the parent cloud of the ONC as $100 M_\odot$. From the correlation observed in embedded clusters, between the total mass of the stars in the cluster and the mass of the most massive star in it, the most massive star a clump of mass $100 M_\odot$ can produce is $\approx 3 M_\odot$ (Weidner & Kroupa 2006; Weidner, Kroupa & Bonnel 2010), if we assume a star formation efficiency of 0.3 (Lada & Lada 2003). This value is interestingly close to that for the lowest mass observed to be segregated in the ONC. It may be noted that the above mentioned correlation, seen between the total mass of the stars in a cluster and the mass of the most massive star in it, need not conflict with the observations, either of isolated O stars or, with numerical simulations which suggest that, clusters form by random sampling from a universal IMF with a fixed stellar upper-mass limit (Lamb et al. 2010). In the first instance, these could be 'run aways', and in the second instance, the correlation would still hold, albeit statistically (see also Oh & Kroupa (2012)). The density of the parent cloud of the ONC is only $\sim 2 \times 10^3 \text{ cm}^{-3}$, which in our scenario, is less than the critical density ($\lesssim 10^6 \text{ cm}^{-3}$) required for a bound cluster to form, from a cloud of mass $6700 M_\odot$ (Eqn 10). Thus, in our scenario, the ONC will evolve into an unbound OB Association and this is consistent with the predictions Huff & Stahler (2006) make, regarding the fate of the ONC, from observations of the velocity dispersion of the stars in the cluster (Jones & Walker 1988).

From the expression for the critical mass (Eqn 10), we see that, in our scenario, the lowest mass which will be segregated, depends directly on the total mass of the clump, inversely on the square root of the density of the clump and, will be smaller, for smaller speeds of random motions of the gas. This last dependence, on the level of small scale turbulence, comes from the fact that gas dynamical friction can slow condensations down to the sound speed (here the speed of small scale turbulence), and the lower the speed of the condensations the smaller will be the two body relaxation time (see Eqn 12 below). The above expectations from our theory, are borne out by observations (Schmeja, Kumar & Ferreira 2008; Chavarria et al. 2010), wherein, the most centrally condensed and mass segregated clusters are found in clouds with the lowest Mach number and vice versa. By making a comparison between the age of the cluster and the migration time scale for the stars, Chavarria et al. (2010) attribute, the mass segregation present in the embedded clusters observed by them, to gas dynamical friction. The densities of these star forming clumps (which are $\gtrsim 10^4 \text{ cm}^{-3}$), are less than the critical densities (Eqn 9) estimated for their masses (which are $\gtrsim 5 \times 10^3 M_\odot$), and we find that, within them, stars with masses $\approx 1 M_\odot$ do not show any concentration towards the center. We do not expect any of the four clumps observed by Chavarria et al. (2010) to produce bound clusters. However the clump DR21(OH) observed by Bontemps et al. (2011), with a mass of $7000 M_\odot$ and a radius of 0.3 pc has a critical mass of only $4.2 M_\odot$ and, we consider it highly probable that this cloud will eventually produce a bound cluster.

5. Discussion

In our scenario, clusters with large masses, could form at the centers of sufficiently dense molecular clouds, by the congregation of the more massive clumps in it, due to deceleration by gas dynamical friction. Observationally some molecular clouds as well as Giant Molecular Clouds, do show, such dense, cluster-bearing concentrations of gas in them (Wilking & Lada 1983; Maíz-Apellániz 2001), and Wilking & Lada (1983) comment on their probable significance in the formation of bound stellar clusters. Also, gas that harbors clusters, is seen to be more dense (Kauffmann et al. 2010) and more highly clumped or substructured than gas that does not bear a cluster (Lada & Lada 2003). In the present scenario, the cluster will bear the imprint of the merged clumps and cores, as substructure in its stellar distribution, till it completely relaxes. This conforms with what is observationally seen, since very young, ($< 1 \text{ Myr}$) star clusters, seem to show significant levels of substructure (Lada & Lada 2003) and, observations show that cores and young clusters have substructure and are cool (sub-virial) (see for example (Schmeja, Kumar & Ferreira 2008; Kirk, Johnstone & Tafalla 2007)). This is consistent also, with the observed difference, between the fractal dimensions of young clusters with internal substructure (that has still not been erased by dynamical relaxation) and, that of molecular gas, and we note that, a more clustered distribution of the densest star forming gas, within the parent molecular clouds – as expected in our scenario – has already been suggested as a probable reason for the above difference in fractal dimensions (Alfaro & Sanchez 2011). Mergers, need not erase any mass segregation that was originally present within the clumps, since dynamical simulations have shown that, in the merging of clusters, which are themselves mass segregated, the mass segregation will be retained, rather than get erased in the merger (McMillan, Vesperini & Portegies Zwart 2008). In this context, it may be interesting to note that the comparatively less dense Taurus complex, does not show any central concentration

or mass segregation.

The critical densities above which we expect the formation of bound clusters / mass segregation, are consistent with the inter-clump medium densities, for which Gorti & Bhatt (1995) notice mass segregation in their numerical simulations studying the effect of gas dynamical friction on prestellar clumps and, Saiyadpour, Deiss & Kegel (1997) obtain bound clusters in their analysis. The larger densities, which in our scenario are a prerequisite for the formation of bound clusters, are also consistent with the general conclusion that the production of a bound cluster must require special physical conditions, to account for their rarity; only about 4 – 7% of all embedded clusters seem to give rise to bound clusters (Lada & Lada 2003). Also Kauffmann et al. (2010) report that, between clouds which have the same radii, those which harbor clusters are more massive than those which do not show a concentration of stars. They also report that while clouds do seem to follow a general mass -size relation of the form $M_c(r) = 400M_\odot r_{\text{pc}}^{1.7}$, there are outliers whose masses (and hence densities) are an order of magnitude larger. In our scenario, such denser clouds could give rise to bound clusters. Goddard, Bastian & Kennicutt (2010) in a study, on the fraction of clusters that escape infant mortality find that, in the galaxies they studied, this fraction is strongly proportional to the Star Formation Rate (SFR) density of the parent galaxy. Since the SFR density is related to the surface gas density by the Kennicutt-Schmidt law, they conclude that the survival fraction of clusters is strongly dependent on the surface gas density. Since we expect clouds, and hence clumps, in regions of larger surface gas densities to be denser, this is consistent with the expectation from our scenario. Our model is consistent also with the observed correlation, that the two categories of massive clusters – bound or ‘starburst’ and unbound or ‘leaky’ – noticed by Pfalzner (2009) obey; that bound clusters are found in regions of high density viz. the Galactic Center or in the spiral arms, and unbound ones in the lower density areas (Pfalzner 2011). She even suggests, as a hint for the origin of the two categories, this obvious difference in their locations. It is also interesting to note that while, among these clusters, the young, bound ones have stellar densities $> 10^5 \text{ cm}^{-3}$, the initial stellar densities for the young, unbound ones are $< 5 \times 10^3 \text{ cm}^{-3}$. The high initial densities quoted for the bound ‘starburst’ clusters – $\sim 10^6 - 10^7 \text{ cm}^{-3}$ – (Pfalzner 2009; Pfalzner & Eckart 2009) are consistent with the densities expected for the compact clusters, that in our scenario, should form in clouds with densities greater than the critical densities we obtain (Eqn 9) (see also Maíz-Apellániz (2001)).

The gas dynamical friction time scale is inversely proportional to the mass of the body which is being decelerated by it. Hence it is possible that, relative segregation, between the stars / cores / clumps of various masses may occur and, some signature of this might survive, even after relaxation, in the clusters evolving out of dense gas clouds. We note that, the brown dwarfs associated with a cluster are distributed in a region much larger than what is occupied by the cluster itself (Luhman 2006; Kumar & Schmeja 2007; Kouwenhoven, Brown & Kaper 2007), and clusters do seem to have associated with them a corona of low mass stars ((Sharma et al 2006; Luhman 2006; Kumar & Schmeja 2007; Kouwenhoven, Brown & Kaper 2007); see also Gorti & Bhatt (1996)). Such a ‘core-halo’ structure has been noticed for some Massive Young Clusters too (Maíz-Apellániz 2001).

It is seen from numerical simulations that, clusters that are dynamically cool at birth can remain bound, down to SFE’s ~ 0.05 , if they lose the gas while they are still cool (Goodwin 1997, 2009; Smith et al. 2011) and that, clusters that are cool and/or have substructure, can mass segregate in a very short time scale (Allison et al. 2009, 2010; Maschberger & Clarke 2011; Olczak, Spurzem & Henning 2011). However, the study by Pelupessy & Portegies Zwart (2012), which extended these studies by including the potential due to the gas, found that, only a remnant with an IMF that was possibly skewed, rather than a mass segregated cluster, is left after gas expulsion. That bound clusters may be obtained, by starting with ‘cool’ initial conditions, and losing the gas before the stars acquire too high speeds due to collapse, may be expected from the fact that, between being born with a very low velocity dispersion, and acquiring virial speeds by collapsing within the gravitational potential of the gas cloud – within about a dynamical time, which is \sim half a million year, for a density $\sim 10^4 \text{ cm}^{-3}$ – (Lada, Margulis & Dearborn 1984)), the stars can, for a short time, satisfy the condition for binding – that the total energy of the stars, taken alone, be negative. However, this would imply that the survivability of clusters would be proportional to the dynamical time, which is inversely proportional to the square root of the density. This would produce an anti-correlation between the cluster survivability and the gas density, which is counter to observations, as explained above (Goddard, Bastian & Kennicutt 2010).

The scenarios mentioned above, which produce fast mass segregation, may be understood by noticing the following. The formula for the two body dynamical relaxation time for a stellar cluster, whose stars are moving with virial speeds (Binney & Tremaine 2008), has to be modified when we consider a cluster that is embedded in gas. With ϵ equal to the ratio of the total mass in stars to the total mass of stars plus gas we get

$$t_{\text{Relax}} = \frac{\alpha^2 N}{8 \ln \alpha N} t_{\text{cross}} \quad (12)$$

where $\alpha = \beta^2 / \epsilon$. ϵ is < 1 for an embedded cluster and, is equal to 1 for a cluster free of gas. The relaxation time would be smaller for – a) larger SFE / gas free clusters b) sub-virial speeds for the stars i.e. $\beta < 1$, and also c) within sub-structure, since in this case, the number of stars as well as the crossing time would be much smaller compared to those for the cluster as a whole. These conclusions are in concurrence with the results of the simulations mentioned above, wherein, the signature of dynamical relaxation viz. mass segregation, is reported to occur fast, in simulations which start with suitable (as per our analysis above) initial conditions. Our model, explores the possibility of gas dynamical friction being the probable cause of the occurrence of ‘suitable’ initial conditions, and also delineates the situations in which such initial conditions might arise.

In our scenario, the difficulty posed by the large dynamical relaxation times in embedded clusters, is avoided by the fact that, gas dynamical friction is capable of producing mass segregation explicitly – due to its mass dependence, as well as implicitly too – by reducing the dynamical relaxation time, through its ability to decelerate condensations to sonic speeds, which in star forming clouds, are subvirial (i.e. by making β go < 1) (Eqn 12). The efficacy of gas dynamical friction may be enhanced, under the following conditions. In turbulent molecular clouds, the decay time for small scale turbulence, will be smaller than that for large scale motions, if $m_t < 3$, where $E_t(k_t) \propto k_t^{-m_t}$ is the energy density of turbulent motions per unit wave number range, k_t being the wave number. For example, for a Kolmogorov spectrum of turbulence, $m_t = 5/3$. Hence the sound speed, which is here taken as the typical speed of small scale turbulence in the cloud, can decrease over time and go over to the actual speed of sound for quiescent gas, in situations where the turbulence is neither forced nor fed, and clumps and cores, which are slowed down to sonic speeds by gas dynamical friction, can be quite sub-virial. This will reduce the relaxation time. The dying down of turbulence can cause the friction to increase also, by allowing longer wakes to develop. Also, for a cloud with a power law density gradient $\propto r^{-m_d}$ ($m_d < 3$), between a small inner radius within which the density is a constant, and a large outer radius beyond which the density may be considered negligible, we get $M_c(r) < \rho(r) >^{-1/2} \propto r^{3-\frac{m_d}{2}}$, $< \rho(r) >$ being the mean density within a radius r . Thus the gas dynamical friction time scale is likely to be smaller than the global mean value towards the centres of real clumps and clouds, as the product $M_c \rho^{-1/2}$, which occurs within the equation for t_{gdf} (Eqn. 4), decreases with r .

Though gas dynamical friction can cause decay of binary orbits (Gorti & Bhatt 1996a) and, binaries and stellar clusters can have significant effects on each other (Kroupa 1995; Kroupa, Petr & McCaughrean 1999; Delgado-Donate, Clarke & Bate 2003; Parker, Goodwin & Allison 2011), we do not consider the possible role, that either binaries or tidal fields (see for example Ballesteros-Paredes et al. (2009)), could play in the formation of bound clusters, as they lie beyond the scope of this paper. The model avoids the need, to take recourse to density dependent IMF's or SFE's (Kruijssen 2012), in accounting for various aspects of stellar clusters. A comparison with such models will be made in a future work. The model offers the following interesting possibility for the formation of Globular Clusters – by the contraction and coalescence, in a similar scenario, of dense cores, within clumps, within clouds, within very massive, very dense molecular clouds, that perhaps formed via turbulent fragmentation during galaxy formation (see also Maíz-Apellániz (2001)).

6. Summary

Gas dynamical friction can decelerate masses and also explicitly as well as implicitly, promote mass segregation. Comparing the gas dynamical friction time-scale for sub-structures within star forming clouds, with the time available before gas expulsion takes place from an embedded stellar cluster, we obtain a boundary value for the density of a star forming clump of given mass, such that, stellar clusters born in clumps which have densities higher than this, could emerge bound after gas loss, due to the locally elevated SFE. For a clump of given mass and density, we find a critical mass such that, subcondensations with larger masses than this could suffer significant segregation within the clump. We compare our results with observations and also other work in the field, and also discuss the implications of our scenario for the formation of bound clusters, vis-a-vis observations.

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